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Department of Mathematics and Statistics
American University of Sharjah
Final Exam - Spring 2017
MTH 111 – ~~Linear Algebra~~

Math for architects.

Date: Thursday May 11, 2017 **Time:** 2:00 pm - 4:00 pm

Student Name	**Student ID Number**
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Instructor Name
Dr. Ayman Badawi

***Do not open** this exam until you are told to begin.*

1. *No questions are allowed during the examination.*
2. *This exam has **11** questions.*
3. *Do not separate the pages of the exam.*
4. *Scientific calculators are allowed.*
5. *Turn off all cell phones and remove all headphones.*
6. *Take off your cap.*
7. *No communication of any kind is allowed during the examination*
8. *If you are found wearing a smart watch or holding a mobile phone at any point during the exam then it will be considered an academic violation and will be reported to the dean's office.*

Student signature: Olive Malik

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Final Exam, MTH 111, Fall 2016

Ayman Badawi

QUESTION 1. (8 points)

(i) $\int (x^2+4)^2\, dx =$

$\int (x^2+4)(x^2+4)\, dx.$

$\int x^4+4x^2+4x^2+16$

$\int x^4+8x^2+16\, dx$

$\int x^4+8x^2+16\, dx.$

$\boxed{= \frac{x^5}{5}+\frac{8x^3}{3}+16x+C.}$

(ii) $\int (x+1)(x^2+2x+1)^{10}\, dx$

Power formula $a\,f'(x)\,(f(x))^{n-1}$ $\quad n-1=10$, $n=11.$

$a\,f'(x) = x+1$

$f'(x) = 2x+2$ $\quad 11a(2x+2) = x+1$

$22a\,\cancel{(x+1)} = \cancel{(x+1)}$

$22a = 1$

$a = 1/22.$

$\boxed{\frac{1}{22}(x^2+2x+1)^{11}+C}$

(iii) $\int (x+1)e^{(2x^2+4x)}\, dx =$

$a e^{f(x)} \rightarrow a\,f'(x)\,e^{f(x)}$ ~~the~~

$f'(x) = (2\times 2)x+4$

$= 4x+4.$

$(4x+4)a = (x+1)$

$4\cancel{(x+1)}a = \cancel{(x+1)}$

$4a = 1$

$a = \frac{1}{4}$ $\longrightarrow$ $\boxed{\frac{1}{4}e^{2x^2+4x}+C}$

(iv) $\int \frac{6x+6}{3x^2+6x-7}\, dx =$

$\frac{a}{\ln B} \times \frac{f'(x)}{f(x)} = \frac{a\,f'(x)}{f(x)}$

$f'(x) \rightarrow (3\times 2)x+6 = 6x+6$ hence $a=1$

$\boxed{\ln\left|(3x^2+6x-7)\right|+C}$

QUESTION 2. (8 points). Find y' and do not simplify

(i) $y = \frac{1+x^2+x^3}{x^{12}}$

$y = (1+x^2+x^3)(x^{-12})$

$y' = (2x+3x^2)(x^{-12}) + (1+x^2+x^3)(-12x^{-13}) \rightarrow \boxed{y' = -12x^{-13}-10x^{-11}-9x^{-10}}$

(ii) $y = e^{(6x^2+7x+1)} + 10x^2 - x + 23$

$y' = e^{6x^2+7x+1} \times ((6\times 2)x+7) + (10\times 2)x - 1$

$\boxed{y' = (12x+7)e^{6x^2+7x+1} + 20x - 1}$

(iii) $y = (21+3x-4x^3)^{10}$

$y' = 10(21+3x-4x^3)^9(3-(4\times 3)x^2) \longrightarrow y' = 10(21+3x-4x^3)^9(3-12x^2)$

(iv) $y = ln[(4x+3)^6(-5x+30)^8]$

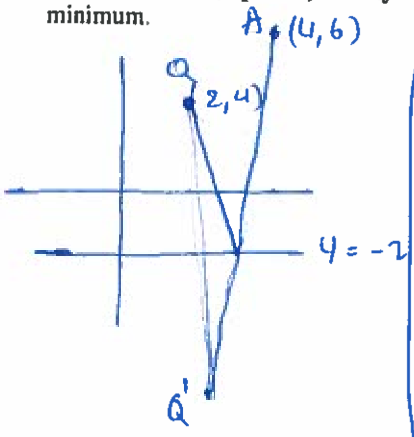
$y = \ln(4x+3)^6 + \ln(-5x+30)^8$

$y = 6\ln(4x+3) + 8\ln(-5x+30)$

$y' = \frac{6\times 4}{4x+3} + \frac{8\times -5}{-5x+30}$

$y' = \frac{24}{4x+3} + \frac{-40}{-5x+30}$

QUESTION 3. (4 points). Let $Q = (2, 4)$, $A = (4, 6)$. Find a point B on the line $y = -2$ such that $|QB| + |AB|$ is minimum.



$Q' \rightarrow 4-(-2) = 4+2 = 6$

$-2-6 = -8.$

$\rightarrow (2, -8)$

Equation of a line.

$(4,6)\ (2,-8)$

$\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$

$\dfrac{y+8}{6+8} = \dfrac{x-2}{4-2}$

$\rightarrow \dfrac{y+8}{14} \times \dfrac{x-2}{2}$

$2(y+8) = (x-2)14$

$2(-2+8) = (x-2)14$

$\dfrac{12}{14} + 2 = x$

$x = 20/7$

Point co-ordinates

$\dfrac{20}{7}$

$\left(\dfrac{20}{7}, -2\right)$

QUESTION 4. (4 points). For what values of x does the tangent line to the curve $y = 4e^{(3x)} - 26x + 2$ have slope equal 10?

$y' = 10$

$y' = (4e^{3x} \times 3) - 26$

$= 12e^{3x} - 26$

$10 = 12e^{3x} - 26$

$\dfrac{10+26}{12} = e^{3x}$

$3 = e^{3x} \longrightarrow \log_e 3 = 3x \rightarrow \ln 3 = 3x.$

$\dfrac{\ln 3}{3} = x = 0.366$

QUESTION 5. (6 points). The plane $P_1 : 2x + 2y - z = 2$ intersects the plane $P_2 : -x + y + 2z = 7$ in a line L. Find a parametric equations of L.

$N_1 = \langle 2, 2, -1 \rangle$

$N_2 = \langle -1, 1, 2 \rangle$

$N_1 \times N_2$

$\begin{vmatrix} 2 & 2 & -1 \\ -1 & 1 & 2 \end{vmatrix} \rightarrow \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} i - \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} j + \begin{vmatrix} 2 & 2 \\ -1 & 1 \end{vmatrix} k$

$[(2\times2) - (-1\times1)]\, i - [(2\times2) - (-1\times-1)]\, j + [(2\times1) - (2\times-1)]\, k$

① $5i - 3j + 4k \rightarrow \langle 5, -3, 4 \rangle$

② Take $z = 0$.

$2x + 2y = 2$

$(-x + y = 7)\, 2$

$-2x + 2y = 14$

$\cancel{2x} + 2y = 2$

$-\cancel{2x} + 2y = 14$

$4y = 16$

$y = \dfrac{16}{4}$

$= 4$

$2x + 2(4) = 2$

$2x + 8 = 2$

$2x = 2 - 8$

$x = \dfrac{-6}{2}$

$x = -3$

$(-3, 4, 0)$

③ Parametric equation is

$(-3, 4, 0) + t\langle 5, -3, 4 \rangle$

$x = -3 + 5t$

$y = 4 - 3t$

$z = 4t$

QUESTION 6. (8 points). Given $y = x^2 - 8x + 25$

(i) Roughly, Sketch the graph of the given parabola.

$y = x^2 - 8x + 25.$

$y = (x-4)^2 - 4^2 + 25$

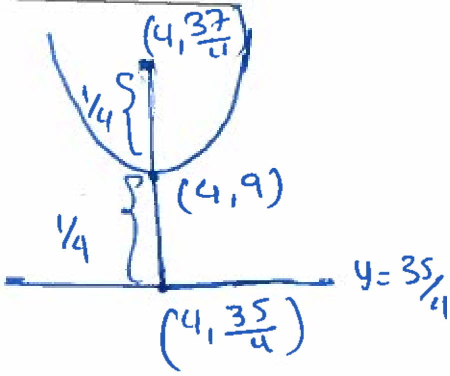
$y + 4^2 - 25 = (x-4)^2$

$(y-9) = (x-4)^2$

$V = (4,9)$

$4d = 1$

$d = \frac{1}{4}$



(ii) What is the directrix line?

$9 - 1/4 = 8.75 = \frac{35}{4}$

$y = \frac{35}{4}$

(iii) What is the focus?

$9 + \frac{1}{4} = \frac{37}{4} = 9.25$

$(4, \frac{37}{4})$

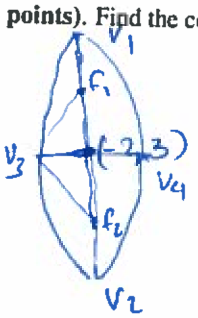
QUESTION 7. (4 points). Find the constant K and the foci of the ellipse $\frac{(x+2)^2}{7} + \frac{(y-3)^2}{16} = 1$

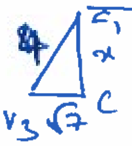
$(\frac{K}{2})^2 = 16$

$\frac{k}{2} = |\sqrt{16}|$

$k = 4 \times 2$

$k = 8$





$\sqrt{4^2 - 7} = 3$

$f_1 = (-2, 3+3) = (-2, 6)$

$f_2 = (-2, 3-3) = (-2, 0)$

QUESTION 8. (4 points). Can we draw the vector $< 6, 1, -2 >$ inside the plane $2x - 6y + 3z = 20$? EXPLAIN

$v \langle 6, 1, -2 \rangle$ $\quad N \langle 2, -6, 3 \rangle$

$(6 \times 2) + (1 \times -6) + (-2 \times 3)$

$= 12 - 6 - 6$

$= 0$ Yes we can because the answer is zero.

QUESTION 9. (3 points).

$$\int_{-6}^{4} f(x) = 7 - 10$$
$$= \underline{\underline{-3}}$$

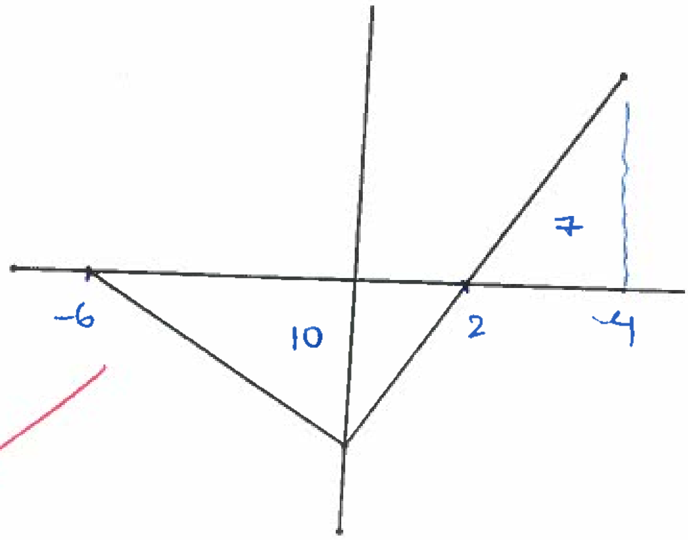


Figure 1. Question: The area of the region that is determined by the curve of f(x) between x = -6 and x = 2 is 10, and the area of the region determined by the curve of f(x) between x = 2 and x = 4 is 7. Find $\int_{-6}^{4} f(x)\,dx$

QUESTION 10. (6 points).

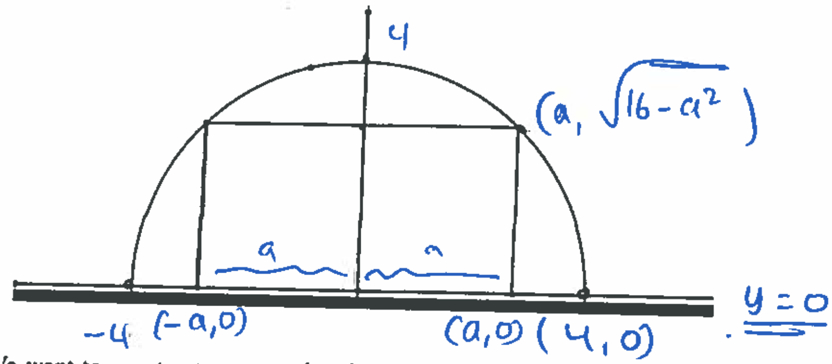


Figure 2. Question: We want to construct a rectangle with maximum area inside the semicircle $y = \sqrt{16 - x^2}$ (see picture). Find the area of such rectangle

$$A = 2a \times (\sqrt{16-a^2} - 0)$$
$$A = 2a(\sqrt{16-a^2})$$
$$A = (2a)(16-a^2)^{1/2}$$

Product formula.

$$A' = 2(16-a^2)^{1/2} + \cancel{2}a \times \tfrac{1}{\cancel{2}}(16-a^2)^{-1/2}(-2a)$$
$$0 = 2(16-a^2)^{1/2} - 2a^2(16-a^2)^{-1/2}$$
$$\cancel{2}(16-a^2)^{1/2} = \cancel{2}a^2(16-a^2)^{-1/2}$$
$$(16-a^2)^{1/2} = a^2(16-a^2)^{-1/2}$$
$$(16-a^2)^{1/2} = \frac{a^2}{(16-a^2)^{1/2}}$$

$$(16-a^2)^{1/2}(16-a^2)^{1/2} = a^2$$
$$16 - a^2 = a^2$$
$$16 = 2a^2$$
$$\pm\sqrt{\frac{16}{2}} = a$$
$$\pm\sqrt{8} = a$$

a is alway +ve

hence

$$a = 2\sqrt{2}.$$

$$A = (2\sqrt{2})2 \times \sqrt{16-(2\sqrt{2})^2}$$
$$= 4\sqrt{2} \times 2\sqrt{2}$$
$$= \underline{16}$$

QUESTION 11. Let $y = -x^3 + 12x + 2$

$y' = -3x^2 + 12$

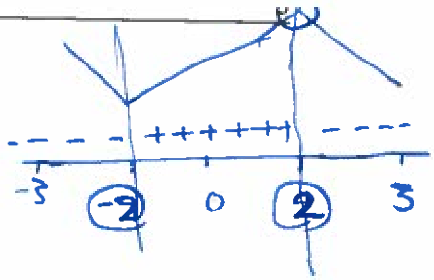
$0 = -3x^2 + 12$

$-12 = -3x^2$

$\pm\sqrt{\frac{12}{3}} = x$

$\pm\sqrt{4} = x$

$\pm 2 = x.$



(i) Find all x values where $f(x)$ is maximum.

$x = 2$

(ii) Find all x values where $f(x)$ is minimum.

$x = -2$

(iii) For what values of x does $f(x)$ increase?

$(-2, 2)$

(iv) For what values of x does $f(x)$ decrease?

$(-\infty, -2) \cup (2, \infty)$

(v) For what values of x do the slopes of tangent lines are positive?

$(-2, 2)$

(vi) What is the equation of the normal line to the curve of f(x) at the point (1, 13)?

$y' = -3x^2 + 12$

$= -3(1)^2 + 12$

$= 9$

negative ceciprocal $= \frac{-1}{9}$

$y = mx + c$

$13 = \frac{-1}{9}(1) + c$

$13 + \frac{1}{9} = c \qquad c = \frac{118}{9}$

$y = mx + c$

$\boxed{y = \frac{-1}{9}x + \frac{118}{9}}$

Faculty information

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